Excel Calculator for Count Times

Overview

The Count Time Calculator is an Excel workbook for estimating the count times needed to achieve specified measurement quality objectives (MQOs) for several types of laboratory radioactivity measurements. Two kinds of MQOs are considered: (1) a required relative method uncertainty at an action level, as defined by the MARLAP manual, and (2) a required detection limit (or minimum detectable concentration) with 5 % Type I and Type II error rates.

There are eight worksheets in the workbook, four of which are used to enter analysis parameters and estimate count times. Each of these four sheets applies to a different type of analysis:

Worksheet	Analysis Types
Basic Analysis	Simple gross counting
AlphaSpec	Alpha-particle spectrometry with alpha-emitting yield tracer
Gross αβ	Simultaneous gross alpha-beta with crosstalk corrections
Gamma	Gamma-ray spectrometry

There are also three sheets that display graphs of analytical performance as functions of varying count times, one each for (1) basic analyses, (2) alpha-particle spectrometry, and (3) gamma-ray spectrometry.

The final worksheet contains lookup tables used by the other sheets (for units of measurement and for radionuclide half-lives and decay constants).

Required Relative Method Uncertainty, φ_{MR}

The *relative method uncertainty* at an analytical action level (AAL) is given by:

$$\varphi_M = \frac{u_c(AC_{AAL})}{AAL}$$

where $u_c(AC_{AAL})$ denotes the combined standard uncertainty of the measured activity concentration when the true activity concentration equals AAL. An MQO can be specified by setting a maximum acceptable value for φ_M. This upper limit is called the *required relative method uncertainty* and is denoted by φ_{MR}.

A measurement meets the required relative method uncertainty when $\varphi_M \le \varphi_{MR}$, or equivalently, when:

$$u_c(AC_{AAL}) \le \varphi_{MR} \cdot AAL$$

Worksheet: Basic Analysis

“Basic” analyses include relatively simple gross counting measurements for single analytes, including measurements of beta-emitting nuclides by gas proportional counting and measurements of either alpha or beta emitters by liquid scintillation. Basic analyses could also include alpha-particle spectrometry that does not involve an alpha-emitting tracer. The parameters for the analysis are entered on the left-hand side of the sheet along with either AAL and φ_{MR} or the required detection limit (MDC_R).

The mathematical model for a basic analysis has the form:

$$AC = \frac{R_S - R_B}{F_1 F_2 \cdots F_n \times DF}$$

where:

AC = activity “concentration” (activity, volumic activity, massic activity, areic activity, …),
R_S = gross sample count rate (C_S / t_S),
R_B = background count rate (C_B / t_B),
F_i = any of several optional divisors, and
DF = decay factor (see below).

The combined standard uncertainty of AC is given by:

$$u_c(AC) = \sqrt{\frac{R_S / t_S + R_B / t_B}{(F_1 F_2 \cdots F_n \times DF)^2} + AC^2 \left(u_{rel}^2(F_1) + \cdots + u_{rel}^2(F_n) + u_{rel}^2(DF)\right)}$$

The MDC is calculated as follows:

$$MDC = \frac{\frac{2.71}{t_S} + 3.29\sqrt{R_B\left(\frac{1}{t_S} + \frac{1}{t_B}\right)}}{F_1 F_2 \cdots F_n \times DF}$$

The sheet includes a table in which you can specify as many as six divisors, F_i, to account for parameters such as the aliquot size, counting efficiency, chemical yield, and radiation emission probability. You can choose an arbitrary name or symbol for each divisor and enter its value and relative standard uncertainty (u_{rel}). If you need fewer than six divisors, leave the remaining rows of the table empty.

A checkbox labeled “Paired background” is provided to allow you to specify that the count time for the background is the same as that for the sample source (e.g., as is common for liquid scintillation counting). If the box is unchecked, you must specify the background count time, t_B. If it is checked, the cell for t_B is locked and protected against editing.

There are two buttons labeled “Calculate.” The first of these calculates the count time, t_S, needed to meet the specified relative method uncertainty, φ_{MR}. The second button calculates the count time needed to meet the required detection limit, MDC_R. After either calculation is completed successfully, the estimated method uncertainty at the AAL and the estimated MDC are displayed on the right-hand side, together with other calculated quantities, and the appropriate MQO cell is highlighted to show which MQO is being achieved. If the “Paired background” box is checked, t_B will be set equal to t_S.

If you click any of the calculated cells, a description of the quantity appears in a small popup window.

Click the “Print” button to generate a Print Preview of the current worksheet, from which you can choose to print to any available printer.

The “Reset” button is provided for situations where a calculation error has occurred in the spreadsheet’s programmed macros. When clicked, it clears certain error conditions, allowing you to continue using the workbook without closing it and reopening it. (Such errors occurred frequently during the development of the workbook but they should be very rare during normal use.) Although this button appears only on the “Basic Analysis” sheet, the error conditions it clears might be produced by any of the four primary work-sheets and can impact the operation of any of those sheets.

Worksheet: AlphaSpec

The AlphaSpec worksheet is used to calculate count times for measurements by alpha-particle spectrometry using an alpha-emitting tracer for chemical yield determination.

The model for alpha-particle spectrometry has the form:

$$AC = \frac{R_{SA} - R_{BA}}{R_{ST} - R_{BT}} \times \frac{c_T \cdot V_T \cdot I_T \cdot DF_T}{V_A \cdot I_A \cdot DF_A}$$

where:

AC = activity concentration,
R_{SA} = analyte gross count rate,
R_{BA} = analyte background count rate,
R_{ST} = tracer gross count rate,
R_{BT} = tracer background count rate,
c_T = tracer massic or volumic activity at its reference date,
V_T = tracer mass or volume,
I_T = alpha emission probability for the tracer,
DF_T = tracer decay factor (see below),
V_A = sample aliquot size,

I_A = alpha emission probability for the analyte, and
DF_A = analyte decay factor (see below).

The chemical yield, Y, is given by:

$$Y = \frac{R_{ST} - R_{BT}}{\varepsilon \cdot c_T \cdot V_T \cdot I_T \cdot DF_T}$$

where ε denotes the counting efficiency.

$$u_c(AC) = \sqrt{F^2 \left(\frac{R_{SA}}{t_S} + \frac{R_{BA}}{t_B} \right) + AC^2 \cdot u_{rel}^2(F)}$$

$$F = \frac{c_T \cdot V_T \cdot I_T \cdot DF_T}{(R_{ST} - R_{BT}) \cdot V_A \cdot I_A \cdot DF_A} = \frac{1}{\varepsilon \cdot Y \cdot V_A \cdot I_A \cdot DF_A}$$

$$u_{rel}^2(F) = \frac{R_{ST}/t_S + R_{BT}/t_B}{(R_{ST} - R_{BT})^2} + u_{rel}^2(c_T) + u_{rel}^2(V_T) + u_{rel}^2(I_T) + u_{rel}^2(DF_T) + u_{rel}^2(V_A) + u_{rel}^2(I_A) + u_{rel}^2(DF_A)$$

The MDC is calculated as:

$$MDC = \frac{\frac{2.71}{t_S} + 3.29\sqrt{R_{BA}\left(\frac{1}{t_S} + \frac{1}{t_B}\right)}}{\varepsilon \cdot Y \cdot V_A \cdot I_A \cdot DF_A}$$

As for a basic analysis, you enter the parameters for the analysis on the left-hand side of the sheet and also either the AAL and the required relative method uncertainty (φ_{MR}) or the required detection limit (MDC_R). Click the appropriate "Calculate" button to calculate the count time needed to meet the specified MQO. After a successful calculation, the appropriate MQO cell is highlighted.

If the count time is not enough to produce at least 500 gross counts for the tracer, the calculated tracer counts are highlighted in red, indicating that either a longer count time or a higher tracer activity is probably needed for reliable quantitation of the analyte, regardless of the estimated method uncertainty.

There is no table of optional divisors for alpha-particle spectrometry, and there is no option for "Paired background." You must specify the background count time, t_B.

You specify the unit of measurement for AC, which can be an activity, massic activity, or volumic activity. You must then specify a unit of compatible type for the sample aliquot size, V_A.

In the same manner, you also specify the unit of measurement for the tracer activity concentration, c_T. You must then specify a unit of compatible type for the tracer amount, V_T.

Click the "Print" button to generate a Print Review, from which you can print to any available printer.

Worksheet: Gross αβ

The Gross αβ worksheet is used to calculate count times for simultaneous gross alpha-beta analyses—e.g., by gas proportional counting or liquid scintillation. The MQOs in this case are the required relative *counting* uncertainties for gross alpha and/or gross beta activity. Only counting uncertainties are considered. No provision is made for required detection limits. Although detection limits could be formally calculated, in practice the estimated limits would likely be unreliable.

The mathematical model for gross αβ is given by:

$$AC_\alpha = \frac{F_{\beta\beta}(R_{S\alpha} - R_{B\alpha}) - F_{\beta\alpha}(R_{S\beta} - R_{B\beta})}{V} \quad \text{and} \quad AC_\beta = \frac{F_{\alpha\alpha}(R_{S\beta} - R_{B\beta}) - F_{\alpha\beta}(R_{S\alpha} - R_{B\alpha})}{V}$$

where:

AC_α, AC_β = alpha and beta activity concentrations,

$F_{\alpha\alpha}$, $F_{\alpha\beta}$, $F_{\beta\alpha}$, $F_{\beta\beta}$ = calibration parameters,
$R_{S\alpha}$ = alpha gross count rate,
$R_{B\alpha}$ = alpha background count rate,
$R_{S\beta}$ = alpha gross count rate,
$R_{B\beta}$ = alpha background count rate, and
V = sample aliquot size.

The parameters $F_{\alpha\alpha}$, $F_{\alpha\beta}$, $F_{\beta\alpha}$, $F_{\beta\beta}$ are derived from the counting efficiencies for each radiation type in each counting window. If ε_{rs} denotes the counting efficiency for radiation r in window s (where $r = \alpha$ or β and $s = \alpha$ or β), then:

$$F_{rs} = \frac{\varepsilon_{rs}}{\varepsilon_{\alpha\alpha}\varepsilon_{\beta\beta} - \varepsilon_{\alpha\beta}\varepsilon_{\beta\alpha}}$$

The combined standard counting uncertainties are given by:

$$u_{cC}(AC_{\alpha}) = \frac{1}{V}\sqrt{F_{\beta\beta}^2\left(\frac{R_{S\alpha}}{t_S} + \frac{R_{B\alpha}}{t_B}\right) + F_{\beta\alpha}^2\left(\frac{R_{S\beta}}{t_S} + \frac{R_{B\beta}}{t_B}\right)}$$

and:

$$u_{cC}(AC_{\beta}) = \frac{1}{V}\sqrt{F_{\alpha\alpha}^2\left(\frac{R_{S\beta}}{t_S} + \frac{R_{B\beta}}{t_B}\right) + F_{\alpha\beta}^2\left(\frac{R_{S\alpha}}{t_S} + \frac{R_{B\alpha}}{t_B}\right)}$$

Enter the analysis parameters on the left-hand side of the sheet. Specify both the alpha and beta activity concentrations, AC_{α} and AC_{β}. Instead of "Calculate" buttons, there are four radio buttons to choose from. If you select the button for "Meet alpha MQO," the value of AC_{α} is treated as the analytical action level for alpha activity, and the value of AC_{β} is treated as an analysis parameter representing an interference in the gross alpha measurement. Similarly, if you select "Meet beta MQO," the value of AC_{β} is treated as the analytical action level for beta activity, and the value of AC_{α} is treated as an analysis parameter representing an interference in the beta measurement. If you select "Meet both MQOs," the spreadsheet performs both calculations and chooses the longer of the two count times it obtains. You can also select the fourth button, "Use standard count time," in which case you must specify the count time (at the bottom). The spreadsheet then estimates the relative counting uncertainties for alpha and beta, $\varphi_{M\alpha}$ and $\varphi_{M\beta}$, using the specified count time.

Click the "Print" button to generate a Print Preview.

Worksheet: Gamma

The Gamma worksheet is used to calculate count times for measurements by gamma-ray spectrometry based on a single photopeak with no interfering peaks and no background-subtraction peak. The mathematical model is:

$$AC = \frac{R_P - R_B}{V \cdot \varepsilon \cdot I_{\gamma} \cdot DF}$$

where:

AC = activity concentration,
R_P = gross count rate in the photopeak region,
R_B = baseline count rate in the photopeak region,
V = sample aliquot size,
ε = counting efficiency,
I_{γ} = gamma-ray intensity, and
DF = decay factor (see below).

Enter the analysis parameters on the left-hand side of the worksheet along with either the analytical action level (AAL) and the required relative method uncertainty (φ_{MR}) or the required detection limit (MDC_R). Click the appropriate "Calculate" button to calculate the count time for the specified MQO.

In practice, the baseline count rate, R_B, is typically estimated from counts observed in several baseline channels on either side of the photopeak. The calculator assumes:

$$R_B = \frac{C_{BL}}{t_S} \cdot \frac{n_P}{n_{BL}}$$

where:

C_{BL} = total counts observed in the baseline channels,
t_S = sample count time,
n_P = number of channels in the photopeak region, and
n_{BL} = number of baseline channels used.

The standard uncertainty of R_B is then given by:

$$u(R_B) = \sqrt{\frac{C_{BL}}{t_S^2} \cdot \frac{n_P^2}{n_{BL}^2}} = \sqrt{\frac{R_B}{t_S} \cdot \frac{n_P}{n_{BL}}}$$

Since the numbers n_P and n_{BL} affect the uncertainty and MDC calculations, you must enter them as analysis parameters. Their ratio is particularly important for estimating the MDC.

The combined standard uncertainty of AC is given by:

$$u_c(AC) = \sqrt{\frac{R_P / t_S + (R_B / t_S)(n_P / n_{BL})}{(V \cdot \varepsilon \cdot I_\gamma \cdot DF)^2} + AC^2\left(u_{rel}^2(V) + u_{rel}^2(\varepsilon) + u_{rel}^2(I_\gamma) + +u_{rel}^2(DF)\right)}$$

The MDC is given by:

$$MDC = \frac{\frac{2.71}{t_S} + 3.29\sqrt{\frac{R_B}{t_S}\left(1 + \frac{n_P}{n_{BL}}\right)}}{V \cdot \varepsilon \cdot I_\gamma \cdot DF}$$

As usual, after the count time is successfully calculated, the appropriate MQO cell is highlighted. You can then click the “Print” button to open a Print Review.

Decay Factors

Wherever a total decay factor, DF, appears on the Basic Analysis, AlphaSpec, or Gamma sheets, it always includes a factor for decay *before* counting, denoted by DBC, and a factor for decay *during* counting, denoted by DDC.

$$DF = DBC \cdot DDC$$

Decay before counting is calculated by a simple expression of the form:

$$DBC = e^{-\lambda t_D}$$

where λ denotes the radionuclide’s decay constant and t_D denotes the elapsed decay time before the start of counting. Decay during counting can be given by three theoretically equivalent expressions:

$$DDC = \frac{1 - e^{-\lambda t_S}}{\lambda t_S} = e^{-\lambda t_S / 2} \cdot \frac{\sinh(e^{-\lambda t_S / 2})}{\lambda t_S / 2} = \frac{P(1, \lambda t_S)}{\lambda t_S}$$

where t_S denotes the count time, $\sinh$ denotes the hyperbolic sine function, and P denotes the regularized lower gamma function.[1] Of these expressions, the first is probably best-known but should not be used unless it is certain that λt_S will never be extremely small, since small values of λt_S can lead to severe floating-point cancellation errors in the numerator. The most current versions of Microsoft Excel provide accurate implementations of the $\sinh$ function, making the second expression useful for calculation; however, some older versions use a naïve implementation that also suffers from internal floating-point

[1] No distinction is made between live time and real/clock time.

cancellation errors when λt_S is too small. So, the spreadsheet uses the last expression, involving the regularized lower gamma function.

The relative standard uncertainty of DF due to the uncertainty of the half-life, $T_{1/2}$, is then given by:

$$u_{\text{rel}}(DF) = \left(\lambda t_D - \lambda t_S \frac{ddc'(\lambda t_S)}{ddc(\lambda t_S)} \right) \cdot u_{\text{rel}}(T_{1/2})$$

where:

$$ddc(x) = \frac{P(1,x)}{x} \quad \text{and} \quad ddc'(x) = -\frac{P(2,x)}{x^2}$$

NOTE: In general, the n^{th} derivative of $ddc(x)$ is given by:

$$ddc^{(n)}(x) = \frac{(-1)^n n!}{x^{n+1}} P(n+1,x) .$$

The regularized lower gamma function $P(a,x)$ is calculated using the Excel formula:

GammaDist(x, a, 1, True).

Decay during counting complicates the calculations for Basic Analysis, AlphaSpec, and Gamma considerably, requiring iterative methods to calculate the count times. Without decay during counting, direct solutions for the sample count times would be possible.

Workbook Protection

The workbook and all of its worksheets are protected against accidental modification, but there is no password for unprotecting them. To unprotect the workbook or any of its sheets, go to the “Review” menu and click *Unprotect Workbook* or *Unprotect Sheet*.

The calculation macros automatically unprotect any worksheet as necessary during certain operations and protect it again when the operations are complete.